Checking cases

Cases

Proposition: If n is a natural number, then $1 + (-1)^n(2n - 1)$ is a multiple of 4.

→ If n is even, then $1 + (2n-1)$ is a multiple of 4

If n is odd then $1 - (2n-1)$ is a multiple of 4

n is even, $1 + 2n - 1 = 2n$.

Cases

Proof: Let $K(n) = 1 + (-1)^n(2n-1)$. We consider the cases where n is odd and even separately. When n is even, $K(n) = 1 + (2n-1) = 2n$. Since n is even, $n = 2m$ for some m, and therefore $K(n) = 2(2m) = 4m$. Therefore $K(n)$ is a multiple of 4.

When n is odd, $K(n) = 1 - (2n-1) = 2 - 2n$. Since n is odd, $n = 2m + 1$ for some m, and therefore

$$K(n) = 2 - 2(2m+1) = 2 - 4m - 2 = 4m$$

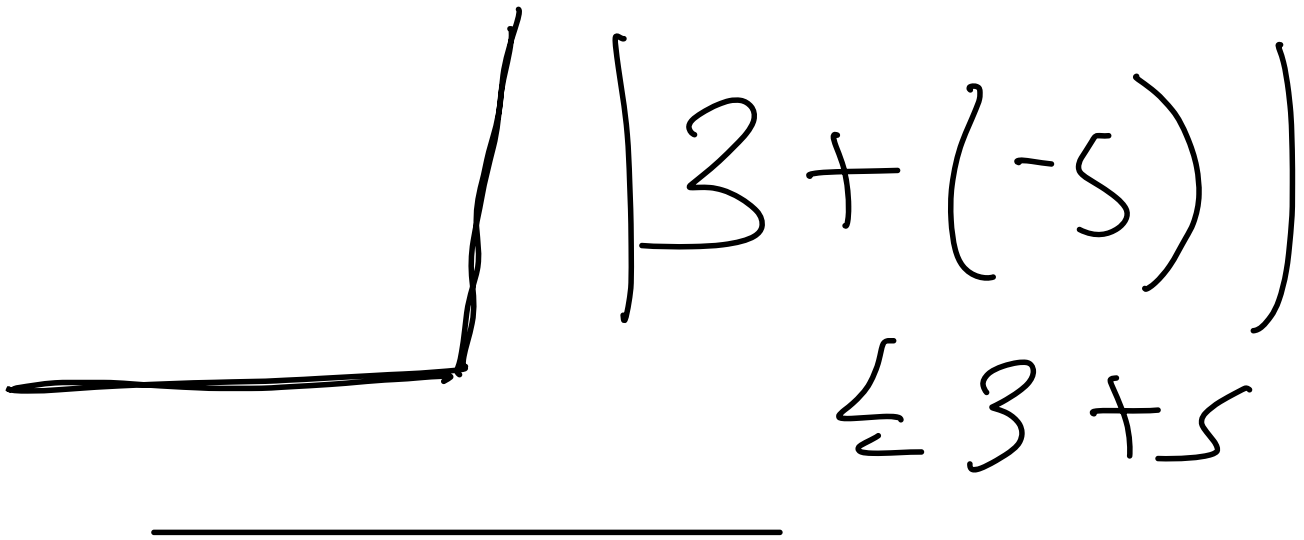
and again $K(n)$ is a multiple of 4.

Cases

Proposition: (The Triangle Inequality) For any real numbers x and y, we have

$$|x + y| \leq |x| + |y|$$

Note: $|x| = x$ if $x \geq 0$, otherwise $|x| = -x$.



Cases

Proof: There are four cases to consider, depending on the signs of x and y, and we take them in turn.

1. $x \geq 0$ and $y \geq 0$. Then $x + y \geq 0$. Therefore, in this case, $|x| = x$, $|y| = y$, and $|x + y| = x + y$ and so $|x + y| = |x| + |y|$.
2. If $x < 0$ and $y < 0$ then $|x + y| = -x - y = |x| + |y|$.

Cases

3. $x \geq 0$ and $y < 0$.

Cases

3. $x \geq 0$ and $y < 0$.

Then $y < x + y < x$. If $x + y \geq 0$, then
$|x + y| = x + y = |x| - |y| \leq |x| + |y|$. If $x + y < 0$, then
$|x + y| = -x - y = -|x| + |y| \leq |x| + |y|$.

The 4th case, $x < 0$ and $y \geq 0$, follows by the same argument.